with the momentum as one would expect to be the case in the absence of the πN $(\frac{3}{2}\,\frac{3}{2})$ isobar and the state $I=1$ of the dipion.

We have determined (Fig. 10) the variation of the branching ratio $U=\sigma(\pi^+p\pi^0)/\sigma(\pi^+\pi^+n)$. No model predicts the observed behavior. However, the slow decrease above 800 MeV/c agrees with the predictions of Olsson and Yodh.[21]

Other remarkable features of our results are the following:

(1). The peak at 1 GeV/c in the cross section $\pi^+p\pi^0$ corresponds to the "shoulder" in the total cross section and is much more marked in this inelastic reaction than in the elastic cross section. The energy is close to the threshold for the reaction $\pi^+p\rightarrow\rho^+p$.

[21] M. Olsson and G. B. Yodh, Phys. Rev. Letters **10**, 353 (1963).

(2). The reactions $\pi^+p\rightarrow\pi^+p\pi^0$ and $\pi^-p\rightarrow\pi^-\pi^+n$ are, among the five reactions of single-pion production, the only ones which contain a pure state with $I=\frac{3}{2}$ of the (πN) system, and, in both of them, an $I=1$ state of the di-pion is possible. The behavior of the two channels is rather similar, in spite of the shift of 200 MeV/c. The corresponding cross sections increase rapidly and attain a value of 10 mb around 1 GeV/c.

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We wish to express our gratitude to the numerous persons who have contributed to this experiment, particularly to G. Valladas for his important part in its conception and to S. Brehin and J. C. Brisson who were in charge of construction and operation of the electronics equipment.

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Electromagnetic Violation of Conservation of Vector Current*

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The electromagnetic violation of vector current conservation is considered and the strong interaction renormalization of the β-decay coupling constant G_V in the presence of electromagnetism is estimated. It is shown that the existing discrepancy between the G_V and the μ-decay coupling constant G_μ may be due to such a renormalization. In this demonstration, use is made of a hypothetical scalar particle with isotopic spin unity. This particle has been used as an auxiliary means and the final results do not depend on the parameters of such a particle. It is shown that this particle has interactions with other particles only if the electromagnetic mass splitting of the isotopic multiplets is not neglected. Other properties of this particle are discussed.

IN the local four-Fermion universal theory of weak interactions, the β-decay coupling constant G_V and that for the μ decay G_μ should be equal in the absence of radiative effects since, according to the conserved vector current hypothesis, there is no renormalization[1] of G_V due to strong interactions when electromagnetism is neglected. But it is well known that there is a discrepancy[2] of about 2% between G_V and G_μ after elimination of the radiative effects. In calculating these radiative effects,[2,3] however, only the simple perturbative diagrams were considered. Thus the change in the strong interaction renormalization of G_V due to radiative effects has not yet been calculated. That is to say, whereas previously in the absence of electromagnetism the whole set of strong interaction graphs which renormalize G_V summed up to unity, now, in the presence of electromagnetism, the strong interaction renormalization factor is no longer unity. Rather, we should expect $G_V=G_\mu+\delta G_V$, where[4] $\delta G_V=O(\alpha)$, α being the fine-structure constant. The purpose of this paper is to estimate δG_V due to the strong interaction renormalization of G_V in the presence of electromagnetism and to see whether such an effect can remove the discrepancy between G_V and G_μ. We shall demonstrate that it is

* Work supported by the U. S. Atomic Energy Commission.
[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[2] L. Durand, III, L. F. Landowitz, and R. B. Marr, Phys. Rev. **130**, 1188 (1963). In this paper, this discrepancy is thoroughly discussed and references to experimental papers and other relevant literature regarding the radiative corrections are given.
[3] R. E. Behrends, R. J. Finkelstein, and A. Sirlin, Phys. Rev. **101**, 866 (1956); S. M. Berman, *ibid.* **112**, 267 (1958); T. Kinoshito and A. Sirlin, *ibid.* **113**, 1652 (1959); S. M. Berman and A. Sirlin, Ann. Phys. (N.Y.) **20**, 20 (1962); N. Chang, Phys. Rev. **131**, 1272 (1963).
[4] R. Behrends and A. Sirlin, Phys. Rev. Letters **4**, 186 (1960) and M. V. Terent'ev, Zh. Eksperim. i Teor. Fiz. **44**, 1320 (1963) [English transl.: Soviet Phys—JETP **17**, 890 (1963)]. These authors have shown that the effect of the mass differences of isotopic multiplets on the G_V is at least second order in the magnitude of the relative mass differences. In their work, however, they essentially considered the effect of the insertion of all electromagnetic self-energy parts in the set of strong interaction renormalization graphs. As has been pointed out by Chang in Ref. 3, electromagnetism has more effects than just self-energy change of the internal charge lines and, therefore, the work of Behrends and Sirlin and Terent'ev does not imply that $G_V-G_\mu\sim G_\mu O(\alpha)$. This will in fact be born out by our calculations.

indeed possible. In our demonstration we shall make use of a hypothetical scalar particle ζ with isotopic spin unity. Whether or not such a particle exists in nature, only experiments can decide. However, our final results will be quite independent of the parameters of such a particle. This ζ particle will be connected with the small violation of conservation of vector current in the presence of electromagnetism and will be shown to have interactions with other particles only if the electromagnetic mass splitting of the isotopic multiplets is not neglected. If such splittings are neglected, then this ζ particle will have no interaction and the vector current will be exactly conserved. In this respect, the ζ particle plays a similar role to that which has been assigned to the κ meson ($I=\frac{1}{2}$, $J=O^+$) by Nambu and Sakurai[5] and Horn[5] in the strangeness violating vector current which will be strictly conserved only if such mass differences as (m_K-m_π) and $(m_\Lambda-m_N)$ tend to zero.

There are, of course, other explanations for the discrepancy between G_V and G_μ. One explanation is that perhaps the electromagnetic correction of virtual W mesons[6] is responsible for the existing μ–β discrepancy. However, if the mass of the W meson is about 1.5 BeV as indicated by recent CERN experiments,[7] such a correction will be too small. Blin-Stoyle and Le Tourneux[8] suggested that effects stemming from a charge dependence of the internucleon potential (which is to be expected from the mass difference of π^+ and π^0 and the difference in the electromagnetic radiative corrections to the $nn\pi^0$, $np\pi^-$, and $pp\pi^0$ vertices) may contribute to the discrepancy between G_V and G_μ. Such effects are probably very small.[9] Anyhow, it is hoped that such effects are included in our calculation. In Cabibo's theory,[10] one starts from the very beginning from nonuniversality and then it is shown that correlation of various experimental data leads to about a 3.3% difference between G_V and G_μ. One, of course, in his approach does not know the deeper dynamical reason for the breakdown of equality between G_V and G_μ. In the recent approach of Marshak, Ryan, Radha, and Raman,[11] there is again no universality in the conventional sense and one gets a difference of about 2.5% between G_V and G_μ because, in their theory, there is an additional source of μ decay. In our approach, an attempt is made to estimate the hitherto uncalculated effects of the strong interactions in the presence of electromagnetism and to see whether such effects are responsible for the discrepancy between G_V and G_μ so that universality in the conventional sense remains between β decay and μ decay. Below we give the details of our approach.

Following the notation of Gell-Mann and Lévy,[12] we write the interaction Lagrangian for β decay and π decay as follows:

$$\mathfrak{L}_{\text{int}}=2^{-1/2}[V_\alpha+P_\alpha]\times[\bar{\nu}\gamma_\alpha(1+\gamma_5)e+\bar{\nu}'\gamma_\alpha(1+\gamma_5)\mu]^+ +\text{H.c.}$$

Now, as stated in the introduction, in the presence of the electromagnetic interaction, the vector current V_α is no longer strictly conserved. We therefore write

$$V_\alpha=V_\alpha^{(1)}+V_\alpha^{(2)},$$

where $\partial V_\alpha^{(1)}/\partial x_\alpha=0$ always, but $\partial V_\alpha^{(2)}/\partial x_\alpha\neq 0$ in the the presence of electromagnetism, i.e. when we do not neglect isotopic mass splittings. Let us now suppose that $\partial V_\alpha^{(2)}/\partial x_\alpha$ is proportional to ζ, where ζ is a $I=1$ scalar field. Then, in the limit of small momentum transfer,

$$\begin{aligned}
\langle p|V_\alpha^{(1)}|n\rangle &\to G_\mu i\bar{U}_p\gamma_\alpha U_n,\\
\langle p|V_\alpha^{(2)}|n\rangle &\to \delta G_V i\bar{U}_p\gamma_\alpha U_n,\\
\langle\pi^0|V_\alpha^{(1)}|\pi^+\rangle &\to 2^{1/2}G_\mu(p_++p_0),\\
\langle\pi^0|V_\alpha^{(2)}|\pi^+\rangle &\to 2^{1/2}\delta G_V(p_++p_0),\\
\langle 0|V_\alpha^{(1)}|\zeta\rangle &=0,\\
\langle 0|V_\alpha^{(2)}|\zeta\rangle &=f_\zeta q_\alpha,
\end{aligned}\tag{1}$$

where $q^2=-m_\zeta^2$ and f_ζ is the decay constant for the leptonic decay of the ζ particle. The effective coupling constant for the β decay is then given by

$$G_V=G_\mu+\delta G_V.$$

Our attempt now is to estimate δG_V. First, we relate it to f_ζ. Note that the ζ particle undergoes leptonic decay only through small violation of the conserved vector current, i.e., through $V_\alpha^{(2)}$. δG_V can be related to f_ζ by a Goldberger-Trieman type of relation.[13] Following the procedure of Ref. 12 or of Bernstein, Fubini, Gell-Mann, and Thirring,[14] we get

$$f_\zeta=(\delta m_N/m_N)m_N(\delta G_V/\sqrt{2}g_{\zeta NN}),\tag{2}$$

[5] Y. Nambu, and J. Sakurai, Phys. Rev. Letters **11**, 62 (1963); D. Horn, Nuovo Cimento **29**, 571 (1963).

[6] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957); R. A. Shaffer, *ibid.* **128**, 1452 (1962).

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[9] L. B. Okun, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* edited by J. Prentki (CERN, Geneva, 1962); A. Altman and W. M. MacDonald, Nucl. Phys. **35**, 593 (1962).

[10] N. Cabibio, Phys. Rev. Letters **10**, 531 (1963).

[11] R. E. Marshak, C. Ryan, T. K. Radha, and R. Raman, Phys. Rev. Letters **11**, 396 (1963).

[12] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1906).

[13] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[14] J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960). See also Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

If ζ is of mass $4m_\pi$ to $6m_\pi$ and we follow the method of Bernstein *et al.*, then although ζ is far removed from the two-particle intermediate-state threshold, but we take the usual point of view that whenever there is a reonance in the two-particle state, then that dominates and the two-particle branch cut is neglected. The only justification for this is that this approach has been successful in many processes, e.g., nucleon-nucleon scattering. Moreover in the method of Gell-Mann and Levy, we do not have this difficulty when we take divergence of the nonconserved part of current proportional to ζ field; but then one has to justify that

$$d(0)F(0)\approx d(-m_\zeta^2)F(-m\zeta^2)=1.$$

if we consider the β decay of the neutron; and

$$f_\zeta=(\delta m_\pi^2/m_\pi^2)m_\pi(\sqrt{2}\delta G_V/g_{\zeta\pi\pi}), \tag{3}$$

if we consider $\pi^+\to\pi^0+l^++\nu$. Here $\delta m_N=m_n-m_p$ and $\delta m_\pi^2=m_{\pi^+}^2-m_{\pi^0}^2$ and our definitions of the coupling constants $g_{\zeta\pi\pi}$ and $g_{\zeta NN}$ correspond to the equivalent Lagrangian densities

$$g_{\zeta\pi\pi}m_\pi\zeta^+\pi^-\pi^0, \quad \text{and} \quad \sqrt{2}g_{\zeta NN}\zeta\bar{p}n. \tag{4}$$

From (2) and (3), we get at once,

$$2g_{\zeta NN}/g_{\zeta\pi\pi}=\delta m_N/2\delta m_\pi, \tag{5}$$

so that

$$g_{\zeta NN}\approx(1/14)g_{\zeta\pi\pi}. \tag{6}$$

This shows that the interaction of ζ with nucleons is very weak; ζ can decay into two pions through electromagnetic interaction only.

We can thus get δG_V either from (2) or (3) provided that we know f_ζ. f_ζ can be estimated as follows: Let us consider the process $\eta^0\to\pi^++l^-+\bar{\nu}$, which is G-parity violating and can proceed only through electromagnetic violation of the conserved vector current, i.e. through $V^{(2)}$. If we consider that it goes through ζ, we get a relation similar to (3), namely,

$$f_\zeta=\frac{m_\eta^2-m_\pi^2}{m_\pi^2}m_\pi\frac{G'}{g_{\zeta\eta\pi}}, \tag{7}$$

where G' is the weak decay constant for the process $\eta^0\to\pi^++l^-+\bar{\nu}$ and $g_{\zeta\eta\pi}$ is defined to correspond to the Lagrangian density

$$g_{\zeta\eta\pi}m_\pi\eta^0(\boldsymbol{\zeta}\cdot\boldsymbol{\pi}). \tag{8}$$

We now consider a model for $\eta^0\to\pi^++l^-+\bar{\nu}$ as shown in Fig. 1. From this model, we get

$$G'=-\beta_\eta(m_\pi^2/m_\pi^2-m_\eta^2)\sqrt{2}G_V, \tag{9}$$

where β is the strength of the effective Hamiltonian density

$$\beta m_\pi^2\eta^0\pi^0, \tag{10}$$

and the subscript η on β in Eq. (9) indicates that η^0 is on the mass shell. Then from (7) and (9), we get

$$f_\zeta=\sqrt{2}G_Vm_\pi(\beta_\eta/g_{\zeta\eta\pi}). \tag{11}$$

To find $g_{\zeta\eta\pi}$ in terms of $g_{\zeta\pi\pi}$, we take the G parity of ζ to be negative and assume that the process $\zeta^+\to\pi^+\pi^0$ is dominated by the η^0 meson, i.e. we consider the sequence

$$\begin{array}{l}\zeta^+\to\pi^++\eta^0\\ \qquad\qquad\quad\searrow\\ \qquad\qquad\qquad\pi^0,\end{array} \tag{12}$$

where the first step is allowed by strong interactions and the second step is an electromagnetic transition. Then we obtain

$$g_{\zeta\pi\pi}=-g_{\zeta\eta\pi}(\beta_\pi m_\pi^2/m_\eta^2-m_\pi^2), \tag{13}$$

where β_π is defined in (10) with the pion now being on

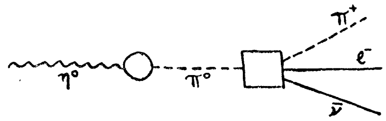

FIG. 1. Feynman diagram for the decay $\eta^0\to\pi^++l^-+\bar{\nu}$ through the π^0.

the mass shell. From (11) and (13) we therefore get

$$f_\zeta=-(\sqrt{2}G_V/g_{\zeta\pi\pi})m_\pi(m_\pi^2/m_\eta^2-m_\pi^2)\beta_\pi\beta_\eta, \tag{14}$$

so that on using Eq. (3), we obtain finally

$$\frac{\delta G_V}{G_V}=-\frac{1}{\delta m_\pi^2/m_\pi^2}\frac{m_\pi^2}{m_\eta^2-m_\pi^2}\beta_\pi\beta_\eta. \tag{15}$$

Our final formula (15) is thus independent of any parameter of the ζ particle and thus this particle has been used only as an auxiliary means.

The constants β_π and β_η appearing in (15) have been estimated by several people. An estimate of β_π was obtained by the present author and Fayyazuddin[15] as

$$\beta_\pi=-\left(\frac{\delta g_{\pi NN}}{g_{\pi NN}}\right)\left(\frac{g_{\pi NN}}{g_{\eta NN}}\right)\left(\frac{m_\eta^2}{m_\pi^2}-1\right), \tag{16}$$

where $\delta g_{\pi NN}$ is the electromagnetic correction to the pion-nucleon coupling constant, i.e., it is the difference between the $pp\pi^0$ and $np\pi^-$ coupling constants since η^0 contributes to the former but not the latter. An estimate of β based on the eightfold way of unitary symmetry[16,17] was obtained by Okubo and Sakita[18] as

$$\begin{aligned}\beta&\approx-(1/\sqrt{3})[(m_{K^0}^2-m_{K^+}^2)+(m_{\pi^+}^2-m_{\pi^0}^2)](1/m_\pi^2)\\ &\approx-0.15.\end{aligned} \tag{17}$$

In this estimate η^0–π^0 transition is taken to be independent of which particle is on its mass shell.

If we now take $\beta_\pi\approx\beta_\eta\approx-0.15$, we obtain from Eq. (15),

$$\delta G_V/G_V\approx-2\%,$$

which is in the right direction and magnitude to account for the existing discrepancy between G_V and G_μ. The value of β given in Eq. (17) predicts a width for

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[15] Riazuddin and Fayyazuddin, Phys. Rev. **129**, 2337 (1963) and **131**, 2839 (1963) (E).

In this paper Eqs. (6) and (7) are in error and should be multiplied by a factor 4. Then using $\lambda/16\pi\approx-0.15$ as pointed out in the erratum, the numbers given for widths of $\eta\to\pi^+\pi^-\pi^0$ and $\eta\to3\pi^0$ in the paper should be multiplied by 64 and those in the erratum by 4 and the branching ratios should be changed accordingly. Our widths (corrected as above) are larger than those obtained in Ref. 18, where η^0–π^0 vertex is estimated by using unitary symmetry model. If, however, we use smaller value of $\delta g_{\pi NN}/g_{\pi NN}$ than previously used, then our estimate for $\eta^0\to\pi^0$ transition agrees with that given in Ref. 18. The coupling constant $g_{\Sigma NK}^2/4\pi$ should also be multiplied by 64 in the paper and by 4 in the erratum. The larger value of pseudoscalar coupling constant $g_{\Sigma NK}^2/4\pi$ thus obtained shows that the K pole in $\Sigma^-\to n+\pi^-$ does not dominate.

[16] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

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[18] S. Okubo and B. Sakita, Phys. Rev. Letters **11**, 50 (1963).

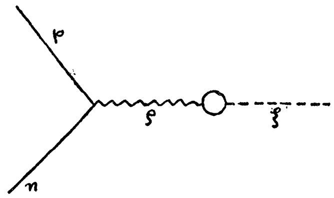

FIG. 2. Feynman diagram for the coupling of the charged ζ with neutron and proton through the ρ meson.

$\eta^0 \to \pi^+\pi^-\pi^0$ of about 142 eV if we consider that the above process goes through the sequence $\eta^0 \to (\pi^0) \to \pi^+\pi^-\pi^0$. However, it has been remarked[19] that at least within the framework of SU_3 if we consider also the sequence $\eta^0 \to \pi^+ + \pi^- + (\eta^0) \to \pi^+\pi^-\pi^0$, then contributions from the above two sequences cancel each other if $\beta_\eta \approx \beta_\pi$. Barrett and Barton[20] have made a dynamical calculation using dispersion relations and including contributions from all possible baryon and antibaryon states. Their calculation allows for the mass dependence of the $\eta^0 \to \pi^0$ "black box." They find, on using eightfold way coupling constants[17] for baryons,

$$\beta_\eta \approx (3 \text{ to } 6)\times 10^{-2}, \quad \text{and} \quad \beta_\pi \approx (1.5 \text{ to } 3)\times 10^{-1}. \tag{18}$$

In this case, our Eq. (15) gives

$$\delta G_V/G_V \approx -0.43\% \text{ to } -1.7\%.$$

The values of β_η and β_π given in Eq. (18) predict a width for $\eta^0 \to \pi^+\pi^-\pi^0$ of 140 to 280 eV.

We have thus shown that it is just possible that the discrepancy between G_V and G_μ may be due to the strong interaction renormalization of G_V in the presence of electromagnetism. In any case, the effect of such a renormalization is not negligible.

We now consider the case where ζ has positive G parity. Then ζ^0 is odd under charge conjugation so that its decay into two pions or two γ's is forbidden and the charged ζ can decay into two pions only via the electromagnetic interaction. Also ζ^0 has no interaction with nucleons since $\zeta^0 \leftrightarrow n+\bar{n}$ and $\zeta^0 \leftrightarrow p+\bar{p}$ are forbidden by charge conjugation invariance and the charged ζ has interaction with the neutron and proton only through G parity violation. In this case, we cannot use the sequence (12) to estimate $g_{\zeta\pi\pi}$ because now we cannot say that the process $\zeta^+ \to \pi^+\pi^0$ is dominated by the η^0 meson [since the first step in (12) now will be also G parity violating]. However, in this case we can estimate $g_{\zeta\pi\pi}$ as follows: Let us consider the coupling of the charged ζ with neutron and proton through the ρ meson as shown in Fig. (2). Then one obtains

$$g_{\zeta NN} = F(\delta m_N/m_\rho^2) g_{1\rho NN}, \tag{19}$$

where our definitions of the coupling constants F and $g_{\zeta NN}$ correspond to the equivalent Lagrangian densities

$$iF\rho_\mu^- \partial_\mu \zeta^+, \quad g_{\zeta^- np}\zeta \bar{p} n = \sqrt{2} g_{\zeta NN} \zeta \bar{p} n.$$

Note that the above model gives no contribution to $g_{\zeta^0 pp}$ or $g_{\zeta^0 nn}$ as it should be. In Eq. (19), $g_{1\rho NN} = \frac{1}{2}\gamma_{\rho NN} = \frac{1}{2}\gamma_{\rho\pi\pi}$, where $\gamma_{\rho NN}$ and $\gamma_{\rho\pi\pi}$ are the same as defined by Sakurai.[21] Hence

$$g_{\zeta NN} = \tfrac{1}{2} F \gamma_{\rho\pi\pi} \frac{\delta m_N}{m_\rho^2} \tag{20}$$

and, on using Eq. (5), we obtain

$$g_{\zeta\pi\pi} = F\gamma_{\rho\pi\pi}(\delta m_\pi^2/m_\pi^2)(m_\pi/m_\rho^2). \tag{21}$$

If we now consider the processes $\pi^+ \to \pi^0 + l^+ + \nu$ and $\rho \to l + \nu$ as going through the conserved part $V^{(1)}$ of the vector current, we get a Goldberger-Treiman type of relation

$$f_\rho = \sqrt{2} G_\mu (m_\rho^2/\gamma_{\rho\pi\pi}),$$

where f_ρ is the decay constant for the process $\rho \to l+\nu$ when it goes through $V^{(1)}$. If we consider the $V^{(2)}$ part of the vector current for these processes, we get a corresponding relation

$$\delta f_\rho = \sqrt{2}\delta G_V (m_\rho^2/\gamma_{\rho\pi\pi}). \tag{22}$$

We now consider the decay $\zeta \to l+\nu$ through the ρ meson as shown in Fig. (3), where of course, ρ decays through δf_ρ since ζ decays only through $V^{(2)}$. Then we obtain

$$f_\zeta = (F/m_\rho^2)\delta f_\rho = F(\sqrt{2}\delta G_V/\gamma_{\rho\pi\pi}), \tag{23}$$

so that, on using Eq. (3), we get

$$F g_{\zeta\pi\pi} = \gamma_{\rho\pi\pi}(\delta m_\pi^2/m_\pi^2) m_\pi. \tag{24}$$

Hence, from (21) and (24), we obtain

$$F^2 = m_\rho^2, \tag{25}$$

$$g_{\zeta\pi\pi} = \gamma_{\rho\pi\pi}(\delta m_\pi^2/m_\pi^2)(m_\pi/m_\rho), \tag{26}$$

and then, on using (20),

$$g_{\zeta NN} = \tfrac{1}{2}\gamma_{\rho\pi\pi}(\delta m_N/m_N)(m_N/m_\rho). \tag{27}$$

In this case, we have not been able to express f_ζ in terms of G_V. We cannot use Eq. (11) for f_ζ because now we cannot express $g_{\zeta\eta\pi}$ in terms of $g_{\zeta\pi\pi}$ through the sequence (12) as remarked already. If, however, we take

$$f_\zeta/f_\pi \approx -\delta m_\pi/m_\pi, \tag{28}$$

where f_π is the decay constant for $\pi^+ \to l+\nu$ and is given by the Goldberger-Treiman relation

$$f_\pi = -G_A \sqrt{2} m_N / g_{\pi NN},$$

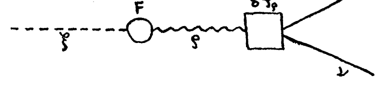

FIG. 3. Feynman diagram for the decay $\zeta \to l+\nu$ through the ρ meson when the latter decays through δf_ρ.

[19] S. Hori, S. Oneda, S. Chiba, and A. Wakasa, Phys. Letters **5**, 339 (1963).

[20] B. Barrett and G. Barton, Phys. Rev. **133**, B466 (1964).

[21] J. J. Sakurai, *Proceedings of the International School of Physics "Enrico Fermi," Varenna* (Academic Press Inc., New York, 1962).

then on using (23), (25), and (28) and[21] $\gamma_{\rho\pi\pi}^2/4\pi \approx 2$, we get

$$(\delta G_V/G_V) \approx -2\%.$$

Thus, if the relation (28) holds, our estimate for $\delta G_V/G_V$ does not appear to depend on the G parity of ζ.

We now consider the ζ particle in more detail. First of all, it is clear from the above discussion that $g_{\zeta\pi\pi}$ vanishes in the limit of charge independence (i.e., when we neglect isotopic mass splittings). For negative G parity of ζ, this is clear from Eq. (13) because β_π, which appears in (13), vanishes in the same limit as is obvious from Eq. (17) or (16). For positive G parity of ζ, this is obvious from Eq. (26). Similar is the case for $g_{\zeta NN}$. Thus the ζ particle exists only in so far as isotopic invariance is violated. Also note that the mass of the ζ particle does not appear in our expressions for the coupling constants. We now give an estimate for the width of the process $\zeta^+ \to \pi^+\pi^0$. It is given by

$$\Gamma_{\zeta^+} = (g_{\zeta\pi\pi}^2/4\pi)(m_\pi^2/4m_\zeta^2)(m_\zeta^2 - 4m_\pi^2)^{1/2},$$

where $g_{\zeta\pi\pi}$ is given by Eq. (13) in the case of negative G parity of ζ and by Eq. (26) for positive G parity. In the former case, $g_{\zeta\eta\pi}$ is a strong interaction coupling constant and hence if we take $g_{\zeta\eta\pi}^2/4\pi \approx 1$ and make use of Eqs. (13) and (17), we get

$$\Gamma_{\zeta^+} \approx 0.75 \text{ to } 0.55 \text{ keV},$$

when m_ζ is $4m_\pi$ to $6m_\pi$. In the latter case, when the G parity of ζ is positive, $g_{\zeta\pi\pi}$ is given by Eq. (26) and, in this case, using $\gamma_{\rho\pi\pi}^2/4\pi \approx 2$, we get

$$\Gamma_{\zeta^+} \approx 3 \text{ to } 2.2 \text{ keV},$$

for the same range of m_ζ as above.

Since the ζ particle is weakly coupled with pions and indeed very weakly coupled with nucleons [see Eq. (6)], it is not surprising that it has not been seen[22] in π–p collisions (particularly due to the large background provided by the ρ meson since the width of the ρ meson is very large) or in p–p collisions. However, for the positive G parity of ζ, it could be produced as a strong process in the model[23] shown in Fig. (4). It may also be possible to look for a ζ resonance in reactions like

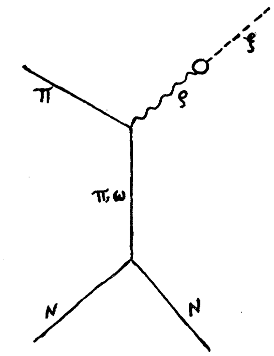

FIG. 4. Feynman diagram for the production of ζ with positive G parity in a strong process.

$$\gamma + p \to n + \zeta^+ \to n + \pi^+ + \pi^0.$$

The ζ particle, if it exists, may provide a dynamical origin of the μ–β discrepancy. It may be that we have universality in weak interactions and that just as the ζ particle may make the effective β-decay coupling constant a little smaller than that for the μ decay in strangeness-conserving leptonic processes, the κ meson considered in Ref. 5 may make the effective coupling constant in strangeness-changing leptonic processes smaller than the universal weak coupling constant. There such an effect is expected to be much larger since mass differences such as $(m_\kappa^2 - m_\pi^2)$ and $(m_\Lambda - m_N)$ are much larger than the mass differences within the isotopic multiplets. This will be in accordance with experimental indications since strangeness-changing leptonic processes are down by an order of magnitude compared to strangeness-conserving leptonic processes.

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[22] For experimental situation regarding this resonance, see talk given by G. Puppi in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* edited by J. Prentki (CERN, Geneva, 1962). For theoretical papers on such a particle, see R. F. Peirels and S. B. Trieman, Phys. Rev. Letters **8**, 339 (1962); and G. Feinberg and A. Pais, *ibid.* **8**, 391 (1962). In these papers other consequences of ζ particles are discussed.

[23] I am indebted to Dr. S. Okubo for pointing this out to me.